2006-2007 Güz Yarıyılı Diferansiyel Denklemler Dersi

Çalışma Soruları 4

Soru1) $L\left(\frac{2}{s^2+3s-4}\right)$ **=?** Laplace dönüşümünü hesaplayınız.

Soru2) $F(s)=\frac{e^{-2s}}{s^2+s-2}$ ters laplace dönüşümünü bulunuz. $L^{-1}[F(s)]=?$

Soru3) $f(t)=(t-3)u_2(t)-(t-2)u_3(t)$ laplace dönüşümü hesaplayın **F(s)=?**

Soru4) $y''+4y'+3y=e^{-2t}$ $y(0)=2$ **,** $y'(0)=-6$ başlangıç değer problemini Laplace dönüşümünden yaralanarak çözünüz.

Soru 5) $y''+4y=\delta(t-4\pi)$ $y(0)=\frac{1}{2}, y'(0)=0$ başlangıç değer problemini çözünüz

Soru 6) $\begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$ diferansiyel denklem sisteminin özdeğerlerini bulunuz

Soru 7) $y'=\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -8 & -5 & -3 \end{bmatrix}y$ diferansiyel denklem sisteminin özdeğerleri ve bu özdeğerlere karşı gelen özvektörleri bulunuz.

Soru 8) $\begin{aligned} \frac{dx_1}{dt} &= x_1-2x_2 \\ \frac{dx_2}{dt} &= -2x_1+x_2 \end{aligned}$ sistemini çözünüz.

Soru9) $x'=\begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix}x+\begin{bmatrix} t^{-3} \\ -t^{-2} \end{bmatrix}$ diferansiyel denklem sisteminin genel çözümünü parametrelerin değişimi metodunu kullanarak bulunuz.

Soru10) $\vec{x}'=\begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}\vec{x}+\begin{bmatrix} e^{-2t} \\ -2e^{t} \end{bmatrix}$ diferansiyel denklem sisteminin genel çözümünü köşegenleştirme metodu kullanarak bulunuz.

Laplace dönüşümü tablosu

	$f(t) = \mathcal{L}^{-1}(F(s))$	$F(s) = \mathcal{L}(f(t))$	
(1)	1	$\frac{1}{s}$,	$s > 0$
(2)	e^{at}	$\frac{1}{s-a}$,	$s > a$
(3)	$\sin(at)$	$\frac{a}{s^2+a^2}$,	$s > 0$
(4)	$\cos(at)$	$\frac{s}{s^2+a^2}$,	$s > 0$
(5)	$e^{bt}\sin(at)$	$\frac{a}{(s-b)^2+a^2}$,	$s > \lvert b\rvert$
(6)	$e^{bt}\cos(at)$	$\frac{s-b}{(s-b)^2+a^2}$,	$s > \lvert b\rvert$
(7)	t^n	$\frac{n!}{s^{n+1}}$,	$s > 0$
(8)	$t^p, p > 0$	$\frac{\Gamma(p+1)}{s^{p+1}}$,	$s > 0$
(9)	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$,	$s > a$
(10)	$f(ct)$	$\frac{1}{c}F(s/c)$	
(11)	$e^{bt}f(t)$	$F(s-b)$	
(12)	$f'(t)$	$sF(s) - f(0)$	
(13)	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$	
(14)	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$	
(15)	$(-t)^n f(t)$	$F^{(n)}(s)$	
(16)	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	
(17)	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$	

1) $\frac{2}{s^2+3s-4}=\frac{2}{(s+4)(s-1)}=\frac{a}{(s+4)}+\frac{b}{(s-1)}$

$a(s-1)+b(s+4)=2$
$s(a+b)=0$
$-a+4b=2$

$a+b=0$
$-a+4b=2$

$a=-\frac{2}{5}, b=\frac{2}{5}$

$$L(\frac{2}{s^2+3s-4})=\frac{-2/5}{(s+4)}+\frac{2/5}{(s-1)}=-2/5e^{-4t}+2/5e^{t}$$

2) $F(s)=\frac{e^{-2s}}{s^2+s-2}$ $\qquad L^{-1}[F(s)]=?$

$\frac{e^{-2s}}{(s-1)(s+2)}\Rightarrow L^{-1}\left[e^{-2s}\frac{1}{(s-1)(s+2)}\right]$

$\frac{1}{(s-1)(s+2)}=\frac{a}{(s-1)}+\frac{b}{(s+1)}$

a(s+2)+b(s-1)=1

s(a+b)=0 , 2a-b=1 a=1/3 ,b= -1/3

$$L^{-1}\left[e^{-2s}\frac{1/3}{(s-1)}-\frac{1/3}{(s+1)}\right]=\left[e^{-2s}(\frac{1}{3}e^{t}-\frac{1}{3}e^{-2t})\right]=u_2(t)\left[\frac{1}{3}e^{(t-2)}-\frac{1}{3}e^{-2(t-2)}\right]$$

$F(s)=\frac{2(s-1)e^{-2s}}{s^2-2s+2}$ $\qquad L^{-1}(F(s))=?$ $\qquad e^{at}\cos bt=\frac{(s-a)}{(s-a)^2+b^2}$

$$L^{-1}(e^{-2s}\frac{2(s-1)}{(s-1)^2+1})=L^{-1}(e^{-2s}2e^{t}\cos t)=2u_2(t)e^{(t-2)}\cos(t-2)$$

$L^{-1}(e^{-cs}F(s))=u_c(t)f(t-c)$

3) $f(t)=(t-3)u_2(t)-(t-2)u_3(t)$ $\qquad$ **F(s)=?**

$(t-3+1-1)u_2(t)-(t-2-1+1)u_3(t)$

$[(t-2)-1]u_2(t)-[(t-3)+1]u_3(t)$

$$e^{-2s}L(t)-\frac{e^{-2s}}{s}-e^{-3s}L(t)-\frac{e^{-3s}}{s}$$

$$\frac{e^{-2s}}{s^2}-\frac{e^{-2s}}{s}-\frac{e^{-3s}}{s^2}-\frac{e^{-3s}}{s} \quad \Rightarrow \quad \frac{e^{-2s}}{s^2}-\frac{se^{-2s}}{s^2}-\frac{e^{-3s}}{s^2}-\frac{se^{-3s}}{s^2}$$

$$\Rightarrow \quad s^{-2}\left[e^{-2s}-se^{-2s}-e^{-3s}-se^{-3s}\right]$$

$$\Rightarrow \quad s^{-2}\left[e^{-2s}(1-s)e^{-3s}(1+s)\right]\text{=F(s)}$$

4) $y''+4y'+3y=e^{-2t}$ $y(0)=2$**,** $y^{'}(0)=-6$ başlangıç değer problemini Laplace dönüşümümünden yaralanarak çözünüz.

Laplace dönüşümünün lineerliğinden

$$L[y'']+4L[y']+3L[y]=L\left[e^{-2t}\right]$$

$L[y'']=s^2Y(s)-sy(0)-y'(0)$
$L[y']=sY(s)-y(0)$ $s^2Y(s)-sy(0)-y'(0)+4sY(s)-4y(0)+3Y(s)$**=1/(s+2)**
$L[y]=Y(s)$

$$s^2Y(s)-2s+6+4sY(s)-8+3Y(s)=\frac{1}{s+2}$$

$$Y(s)\left[s^2+4s+3\right]-2s-2=\frac{1}{s+2}, \qquad Y(s)\left[s^2+4s+3\right]=\frac{1}{s+2}+2(s+1) \quad ,$$

$$Y(s)=\frac{1}{(s+1)(s+2)(s+3)}+\frac{2}{s+3}$$

Basit kesirlere ayrılırsa;

$$\frac{1}{(s+1)(s+2)(s+3)}=\frac{a}{s+1}+\frac{b}{s+2}+\frac{c}{s+3}$$ a= 1/2 , b=-1 , c= ½

Y(s)= $\frac{1/2}{s+1}-\frac{1}{s+2}+\frac{5/2}{s+3}$ bu ifadenin her iki tarafının laplace dönüşümü alınırsa

$$L^{-1}\left(Y(s)\right)=\frac{1}{2}L^{-1}(\frac{1}{s+1})-L^{-1}(\frac{1}{s+2})+\frac{5}{2}L^{-1}(\frac{1}{s+3})$$

$$y(t)=\frac{1}{2}e^{-t}-e^{-2t}+\frac{5}{2}e^{-3t}$$

5) $y''+4y=\delta(t-4\pi)$

Laplace dönüşümünün lineerliğinden **$L(\delta(t-t_0))=e^{-st_0}$**

$$L[y'']+4L[y]=L[\delta(t-t_0)]=e^{-4\pi s}$$

$$L[y'']=s^2Y(s)-sy(0)-y'(0)$$
$$L[y]=Y(s)$$

$$s^2Y(s)-sy(0)-y'(0)+4Y(s)=e^{-4\pi s}$$

$$s^2Y(s)-\frac{s}{2}+4Y(s)=e^{-4\pi s}$$

$$(s^2+4)Y(s)=\frac{s}{2}+e^{-4\pi s}=\frac{s+2e^{-4\pi s}}{2}$$

$$e^{-cs}F(s)=u_c(t)f(t-c)$$

$$Y(s)=\frac{s+2e^{-4\pi s}}{2(s^2+4)}=\frac{s}{2(s^2+4)}+\frac{2}{2(s^2+4)}e^{-4\pi s}$$

Y(S)=1/2 COS2t+1/2sin2t$e^{-4s\pi}$=1/2 COS2t+ $u_{4\pi}$ 1/2sin2(t-4π)

6)

$$\begin{pmatrix}1 & 2 & 1\\ 6 & -1 & 0\\ -1 & -2 & -1\end{pmatrix}$$

$$\det(A-\lambda I)=0$$

$$\begin{pmatrix}1-\lambda & 2 & 1\\ 6 & -1-\lambda & 0\\ -1 & -2 & -1-\lambda\end{pmatrix}$$

$$1-\lambda[(-1-\lambda)(-1-\lambda)]-2[6(-1-\lambda)]+1[-12-(-1*(-1-\lambda))]=0$$

$$\lambda^3 + \lambda^2 - 12\lambda = 0$$

$$\lambda(\lambda^2 + \lambda - 12) = 0$$

$$\lambda_1 = 0 \rightarrow \lambda_2 = 3 \rightarrow \lambda_3 = -4$$

3 farklı reel kök(özdeğerler) elde edilir.

$\lambda_1 = 0$ **için özvektörler**

-6*R$_1$+R$_2$ ve R$_1$+R$_3$ satır işlemleri ile

$$\begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -13 & -6 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{21} \\ x_{31} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x_{11} + 2x_{21} + x_{31} = 0$$

$$-13x_{21} + -6x_{31} = 0$$

$$-13x_{21} = 6x_{31} \rightarrow x_{21} = -\frac{6}{13}x_{31}$$

$x_{31} = 1$ **seçilirse** $x_{21} = -\frac{6}{13}$ $x_{11} = -\frac{25}{3}$

$$\vec{x_1} = \begin{pmatrix} x_{11} \\ x_{21} \\ x_{31} \end{pmatrix} = \begin{pmatrix} -25/3 \\ -6/3 \\ 1 \end{pmatrix}$$

$\lambda_2 = 3$

$$\begin{pmatrix} -2 & 2 & 1 \\ 6 & -4 & 0 \\ -1 & -2 & -4 \end{pmatrix} \begin{pmatrix} x_{12} \\ x_{22} \\ x_{32} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

3*R$_1$+R$_2$ ve -1/2*R$_1$+R$_3$ satır işlemleri ile

$$\begin{pmatrix} -2 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & -3 & -9/2 \end{pmatrix}$$

ve 2/3*R$_3$+R$_2$ ile

$$\begin{pmatrix} -2 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & -3 & -9/2 \end{pmatrix}\begin{pmatrix} x_{12} \\ x_{22} \\ x_{32} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$ **elde edilir.**

$$-2x_{12} + 2x_{22} + x_{32} = 0$$
$$-3x_{22} - \tfrac{9}{2}x_{32} = 0$$

$$x_{22} = -\tfrac{3}{2}x_{32}$$

$$x_{32} = 1$$
$$x_{22} = -\tfrac{3}{2}$$
$$x_{12} = -1$$

$$\overrightarrow{x_2} = \begin{pmatrix} x_{12} \\ x_{22} \\ x_{32} \end{pmatrix} = \begin{pmatrix} -1 \\ -3/2 \\ 1 \end{pmatrix}$$

$\lambda_3 = -4$ **için özvektörler**

$$\begin{pmatrix} 5 & 2 & 1 \\ 6 & 3 & 0 \\ -1 & -2 & 3 \end{pmatrix}\begin{pmatrix} x_{13} \\ x_{23} \\ x_{33} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$ **-6/5R₁+R₂ ve1/5R₁+R₃** $$\begin{pmatrix} 5 & 2 & 1 \\ 0 & 3/5 & -6/5 \\ 0 & -8/5 & 16/5 \end{pmatrix}$$ **1/3*R₂ ,1/8*R₃**

$$\begin{pmatrix} 5 & 2 & 1 \\ 0 & 1/5 & -2/5 \\ 0 & -1/5 & 2/5 \end{pmatrix}$$ **R₂+R₃** $$\begin{pmatrix} 5 & 2 & 1 \\ 0 & 1/5 & -2/5 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_{13} \\ x_{23} \\ x_{33} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$5x_{13} + 2x_{23} + x_{33} = 0$$
$$1/5x_{23} - 2/5x_{33} = 0$$

$$x_{23} = 2x_{33}$$
$$x_{33} = 1 \rightarrow x_{23} = 2 \rightarrow x_{13} = -1$$

$$\overrightarrow{x_3} = \begin{pmatrix} x_{13} \\ x_{23} \\ x_{33} \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

3 farklı reel kök durumunda genel çözüm

$$x_{genel} = c_1\overrightarrow{x_1}e^{\lambda_1 t} + c_2\overrightarrow{x_2}e^{\lambda_2 t} + c_3\overrightarrow{x_3}e^{\lambda_3 t}$$

$$x_{genel} = c_1\begin{pmatrix} -25/3 \\ -6/3 \\ 1 \end{pmatrix} + c_2\begin{pmatrix} -1 \\ -3/2 \\ 1 \end{pmatrix}e^{3t} + c_3\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}e^{-4t}$$

7)

$$y' = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & \_1 \\ -8 & -5 & -3 \end{pmatrix} \vec{y}$$

$$\det(A - \lambda I) = 0$$

$$\begin{pmatrix} 1-\lambda & 1 & 1 \\ 2 & 1-\lambda & -1 \\ -8 & -5 & -3-\lambda \end{pmatrix} = 0$$

$$1-\lambda\left[(1-\lambda)(-3-\lambda)-(-5*(-1))\right]-1\left[(2*(-3-\lambda)-8)\right]+1\left[(2*-5)-(1-\lambda)*-8)\right]=0$$

$$\lambda^3 + \lambda^2 - 20\lambda = 0$$

$\lambda(\lambda^2 + \lambda - 20) = 0$ **3 farklı reel kök(özdeğerler)**

$$\lambda_1 = 0 \rightarrow \lambda_2 = 4 \rightarrow \lambda_3 = -5$$

6. Sorudakine benzer olarak özdeğerlere karşı gelen özvektörler bulunarak genel çözüm elde edilir.

3 farklı reel kök durumunda genel çözüm

$$y_{genel} = c_1 \overrightarrow{y_1} e^{\lambda_1 t} + c_2 \overrightarrow{y_2} e^{\lambda_2 t} + c_3 \overrightarrow{y_3} e^{\lambda_3 t}$$

8) $\dfrac{dx_1}{dt} = x_1 - 2x_2$

$\dfrac{dx_2}{dt} = -2x_1 + x_2$ ***sistemini çözünüz.*** ***x'=Ax homojen sistem***

det(A-λI)=0 oluşturularak özdeğerler:

$\begin{bmatrix} 1-\lambda & -2 \\ -2 & 1-\lambda \end{bmatrix} = \lambda^2 - 2\lambda - 3 = 0$ $\lambda_1 = 3$ $\lambda_2 = -1$ ***(farklı 2 reel kök)***

$\lambda_1 = 3$

$\begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $x_{11} = -x_{21}$

$$\vec{x}_1 = \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

λ_2=-1 için

$$\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \boldsymbol{x_{12}=x_{22}}$$

$$\vec{x}_2 = \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\boldsymbol{x_{homojen}=c_1} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{3t} + \boldsymbol{c_2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t}$$

9)

$x' = \begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix} x + \begin{bmatrix} t^{-3} \\ -t^{-2} \end{bmatrix}$ diferansiyel denklem sisteminin genel çözümünü bulunuz.

Çözüm: x’=Ax+f(t) tipi, önce homojen çözüm yapılır.

x’=Ax det(A-λI)=0 oluşturularak özdeğerler:

$\begin{bmatrix} (4-\lambda) & -2 \\ 8 & (-4-\lambda) \end{bmatrix}$=(4-λ)(-4-λ)+16=$\lambda^2$=0 $\lambda_1 = \lambda_2 = \lambda = 0$ 2 katlı kök

λ_1=0 için

$$\begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$4x_{11}-2x_{21}=0$ $\quad$ $x_{11}=1/2\, x_{21}$ $\quad$ $x_{21}=2$ seçilirse

$$\vec{x}_1 = \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

λ_2=2 için

$$\begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix}\begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \mathit{4x_{12}\text{-}2x_{22}=1} \qquad \mathit{x_{12}=0 \text{ ise}} \qquad \vec{x}_2 = \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ -1/2 \end{bmatrix}$$

Genel Çözüm 2 katlı kök olması durumunda ($\lambda_1 = \lambda_2 = \lambda$)

$$\mathbf{x_{genel}} = \mathbf{c_1}\, \vec{x}_1\, \mathbf{e^{\lambda t}} + \mathbf{c_2}(\vec{x}_1\, t + \vec{x}_2)\mathbf{e^{\lambda t}}$$

idi.

$$\mathbf{x_{homojen}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{c_1}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \mathbf{c_2}\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} t + \begin{bmatrix} 0 \\ -1/2 \end{bmatrix}\right)$$

$$x_1 = c_1 + c_2 t$$
$$x_2 = 2c_1 + 2c_2 t - 1/2c_2$$

Parametrelerin değişimi metodu ile

$$\begin{aligned} x_1 &= u_1 + u_2 t \\ x_2 &= 2u_1 + 2u_2 t - 1/2u_2 \end{aligned} \quad \textbf{(1)} \qquad \text{yazılarak} \quad \Psi = \begin{pmatrix} 1 & t \\ 2 & (2t - 1/2) \end{pmatrix} \text{ olmak üzere}$$

ψ(t) u'(t)=f(t) oluşturulursa

$$u_1' + u_2' t = t^{-3} \qquad -1/2u_2' = -2t^{-3} - t^{-2}$$
$$2u_1' + 2u_2' t - 1/2u_2' = -t^{-2} \qquad u_2' = 4t^{-3} + 2t^{-2}$$

$$u_2 = -2t^{-2} - 2t^{-1} + K_2 \qquad \textbf{(2)}$$

$$u_1' + (4t^{-3} + 2t^{-2})t = t^{-3}$$

$$u_1' = t^{-3} - 4t^{-2} - 2t^{-1} \qquad u_1 = -1/2t^{-2} + 4t^{-1} - 2\ln t + K_1 \qquad \textbf{(3)}$$

(2) ve (3) ,(1) de yerlerine konulursa

$x_1 = K_1 + K_2 t - 1/2t^{-2} + 2t^{-1} - 2\ln t - 2$
$x_2 = 2K_1 + 2K_2 t - K_2/2 - t^{-2} + 8t^{-1} - 4\ln t - 4 - 4t^{-1} + t^{-2} + t^{-1} = 2K_1 + 2K_2 t - K_2/2 + 5t^{-1} - 4\ln t - 4$

veya

$$\mathbf{x}=\mathbf{K}_1\begin{bmatrix}1\\2\end{bmatrix}+\mathbf{K}_2\left(\begin{bmatrix}1\\2\end{bmatrix}t+\begin{bmatrix}0\\-1/2\end{bmatrix}\right)+\begin{bmatrix}-1/2\\0\end{bmatrix}t^{-2}+\begin{bmatrix}2\\5\end{bmatrix}t^{-1}+\begin{bmatrix}-2\\-4\end{bmatrix}\ln t+\begin{bmatrix}-2\\-4\end{bmatrix}$$

10) $\vec{x}' = \begin{bmatrix}1 & 1\\4 & -2\end{bmatrix}\vec{x}+\begin{bmatrix}e^{-2t}\\-2e^{t}\end{bmatrix}$ diferansiyel denklem sisteminin genel çözümünü köşegenleştirme metodu kullanarak bulunuz.

Çözüm:

x’=Ax det(A-λI)=0 oluşturularak özdeğerler:

$$\begin{bmatrix}(1-\lambda) & 1\\4 & (-2-\lambda)\end{bmatrix}=(1-\lambda)(-2-\lambda)-4=\lambda^2+\lambda-6=0$$

$\lambda_1=-3$, $\lambda_2=2$ 2 farklı reel kök

$\lambda_1=-3$

$$\begin{bmatrix}4 & 1\\4 & 1\end{bmatrix}\begin{bmatrix}x_{11}\\x_{21}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$ ***$4x_{11}=-x_{21}$***

$$\vec{x}_1=\begin{bmatrix}x_{11}\\x_{21}\end{bmatrix}=\begin{bmatrix}1\\-4\end{bmatrix}$$

$\lambda_2=2$ için

$$\begin{bmatrix}-1 & 1\\4 & -4\end{bmatrix}\begin{bmatrix}x_{12}\\x_{22}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$ ***$x_{12}=x_{22}$*** $$\vec{x}_2=\begin{bmatrix}x_{12}\\x_{22}\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$$

Özvektörleri içeren matrisi T ile gösterirsek $T=\begin{bmatrix}1 & 1\\-4 & 1\end{bmatrix}$

x=Ty $\longrightarrow$ x′=Ax+f(t) de yerine konursa

$$Ty'=Aty+f(t)$$

olur. Bu denklemin her iki tarafı T^{-1} çarpılarak

$y'=(T^{-1}AT)y+T^{-1}f(t)=Dy+T^{-1}f(t)$ **D=**$\begin{bmatrix}-3 & 0\\0 & 2\end{bmatrix}$

T^{-1} in hesabı

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -4 & 1 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 5 & 4 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/5 & 1/5 \end{array}\right] \longrightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1/5 & -1/5 \\ 0 & 1 & 4/5 & 1/5 \end{array}\right]$$

$\mathbf{T^{-1}} = \begin{bmatrix} 1/5 & -1/5 \\ 4/5 & 1/5 \end{bmatrix}$ **$y' = Dy + T^{-1}f(t)$ oluşturulursa**

$$\mathbf{y'} = \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{bmatrix} 1/5 & -1/5 \\ 4/5 & 1/5 \end{bmatrix} \begin{bmatrix} e^{-2t} \\ -2e^{t} \end{bmatrix}$$

$$y_1' + 3y_1 = \frac{1}{5}e^{-2t} + \frac{2}{5}e^{t}$$

$$y_2' - 2y_2 = \frac{4}{5}e^{-2t} - \frac{2}{5}e^{t}$$ **integrasyon çarpanı yardımıyla**

$$\left[y_1 e^{3t}\right]' = \frac{1}{5}e^{t} + \frac{2}{5}e^{4t} \longrightarrow \quad y_1 e^{3t} = \frac{1}{5}e^{t} + \frac{1}{10}e^{4t} + c_1 \qquad y_1 = \frac{1}{5}e^{-2t} + \frac{1}{10}e^{t} + c_1\, e^{-3t}$$

$$\left[y_2 e^{-2t}\right]' = \frac{4}{5}e^{-4t} - \frac{2}{5}e^{-t} \longrightarrow \; y_2 e^{-2t} = -\frac{1}{5}e^{-4t} + \frac{2}{5}e^{-t} + c_2 \qquad y_2 = -\frac{1}{5}e^{-2t} + \frac{2}{5}e^{t} + c_2 e^{2t}$$

bulunur. Buradan

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} 1/5 \\ -1/5 \end{bmatrix} e^{-2t} + \begin{bmatrix} 1/10 \\ 2/5 \end{bmatrix} e^{t} + \begin{bmatrix} c_1 e^{-3t} \\ c_2 e^{2t} \end{bmatrix}$$

Bu sistemin her iki tarafı T dönüşüm matrisi ile çarpılırsa genel çözüm($x=Ty$)

$$x = Ty = \begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{5}e^{-2t} + \frac{1}{10}e^{t} + c_1 e^{-3t} \\ -\frac{1}{5}e^{-2t} + \frac{2}{5}e^{t} + c_2 e^{2t} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}e^{t} + c_1 e^{-3t} + c_2 e^{2t} \\ -e^{-2t} - 4c_1 e^{-3t} + c_2 e^{2t} \end{bmatrix}$$

veya

$$x = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} e^{t} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} e^{-2t} + c_1 \begin{bmatrix} 1 \\ -4 \end{bmatrix} e^{-3t} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$$